

The University of Chicago

AN INVESTIGATION OF THE MOTION OF PERIODIC COMET D'ARREST (1851 II)

A DISSERTATION SUBMITTED TO THE
FACULTY OF THE DIVISION OF THE PHYS-
ICAL SCIENCES IN CANDIDACY FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS

1939

By

ALBERT W. RECHT

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AN INVESTIGATION OF THE MOTION OF PERIODIC COMET D'ARREST (1851 II)

By ALBERT W. RECHT

SUMMARY.

The work on *Comet Encke* is reviewed. Investigations of the motion of this comet of shortest period by ENCKE, BACKLUND, and others have shown that its motion cannot be adequately represented by applying the law of gravity alone and taking into account all known forces. Instead, investigators have found it necessary to assume an acceleration of the mean motion and even to introduce arbitrary changes in the eccentric angle.

The difficulties experienced in predicting accurately the return of other periodic comets indicates a need for further research along this line. The present investigation takes up the problem of the motion of *Comet d'Arrest*, with a period of about $6\frac{2}{3}$ years, which has been observed at eight appearances since its discovery in 1851. COWELL's method of numerical integration with an interval of ten days is employed to calculate the motion of the comet for the period 1851-1924. Special perturbations are computed for all planets except *Mercury*, whose mass is added to the *Sun*, and for *Pluto*.

Attempts are made to obtain a single system of osculating elements that will represent closely the normal places of all eight apparitions, by combining three appearances in a single least squares solution involving equations of condition. When the results of several such solutions indicate the impossibility of finding a single system for even three appearances, seven new systems of osculating elements are derived which give the best representation possible of the normal places for the pairs of successive appearances.

A comparison of these systems indicates unaccountable systematic changes in the major axis, the eccentricity, and perhaps in the times of perihelion. These indicate an acceleration of the mean motion and eccentric angle of apparently the same order of magnitude for *Comet d'Arrest* as for *Comet Encke*, but opposite in direction.

I. INTRODUCTION.

It has always been difficult to predict closely the time and place of the return of a periodic comet. The differences, often large, between the predicted and actual positions of a returning comet may be the result of one or more of several causes: (a) since the head of most comets is relatively large and diffuse, the individual observations used in making the prediction may be inherently inexact; (b) the nucleus, which is the point usually measured, may be ill-defined, its position may be oscillatory, or the center of gravity may not be the center of light; (c) the short arc over which comets can be observed, while they are in the vicinity of the *Sun* and *Earth*, may be inadequate for a reliable determination of elements; (d) the perturbations between appearances may be computed only approximately; (e) even when special perturbations are computed for all known bodies their masses may not be well determined; or (f) there may be unknown forces which are due to the nature of comets themselves, or which may be a property of the solar system.

To determine which of these factors is responsible for the large discrepancies in comet prediction is a problem worthy of research. A complete solution of this problem will probably require rigorous investigations of the motions of a number of periodic comets. Formerly the computation of special perturbations was a laborious process; but now with the introduction of modern methods of numerical integration which are adapted to the use of calculating machines, and with the publication of *Planetary Coordinates for the Years 1800-1940 Referred to the Equinox of 1950.0* by the Nautical Almanac Office, the labor has been greatly reduced. Without the publication of *Planetary Coordinates* it is doubtful if the present investigation would have been started.

II. PURPOSE.

The purpose of this investigation is to determine whether the motion of *Comet d'Arrest* can be adequately represented by taking into account all known forces, or whether the comet has an accelerated mean motion, such as that of *Comet Encke*. *Comet d'Arrest* is selected because it has been observed at a large number of appearances and has been rather closely followed. It was discovered in 1851, has a period of about $6\frac{2}{3}$ years, and was observed at eight appearances in 1851, 1857-58, 1870, 1877, 1890, 1897, 1910-11, and 1923-24. The returns of 1863, 1883, 1903, 1917, 1930 and 1937 were not observed.

Since this investigation was undertaken in 1934 and *Comet d'Arrest* was to return in the fall of 1936, there was at first the added purpose of making a precise prediction for this unfavorable return to help to prevent the complete loss of the comet if the second successive return were missed. However, it was later found to be impossible to realize this purpose.

III. SIMILAR INVESTIGATIONS.

A. *Comet Encke*. The classic example of this kind of investigation is that of *Comet Encke*. The original theory of the motion of this comet was formulated by ENCKE. BACKLUND reviewed the work of ENCKE and modified the theory to fit later apparitions. He was assisted by VON ASTEN and his work has been carried on since his death in 1916 by MATKIEWICZ. IDELSON has also developed the theory of four recent appearances. All the investigators have found it necessary to assume an acceleration of the mean motion, and some have even introduced changes in the eccentricity to represent the observations accurately.

BACKLUND⁽¹⁾ gives a summary of the theory of the motion of *Comet Encke* from 1819 to 1908. ENCKE found that the acceleration of mean motion was constant, BACKLUND agrees with ENCKE for the period both investigated, but says: "If we represent the mean motion of the comet by the formula

$$n = n_0 + n_1 \tau \quad \left(\tau = \frac{t}{1200} \right)$$

[1200 days is very nearly the period of the comet]

the value of n_1 was very nearly constant during 1819-58 and approximately 0".1. The apparitions 1861, 1865, 1868, gave a value of about ".08 for n_1 , and from 1871 to 1895 only ".062." For calculating the change in the eccentric angle φ BACKLUND used $\varphi = \varphi_0 - 1".96 \tau$.

For the period 1901-14 BACKLUND used $n_1 = ".037$ and

$\varphi_1 = -1".25$. MATKIEWICZ has since used $n_1 = ".046$ and $\varphi_1 = -1".25$ for the period 1918-24 but found it necessary to use $n_1 = ".0204$ for the appearances 1928-1931-1934.⁽²⁾ IDELSON⁽³⁾ was able to represent the four appearances 1924-34 very closely without employing a correction φ_1 and by using $n_1 = ".038$, a value close to that assumed by BACKLUND for the period 1901-14.

According to BACKLUND the acceleration of mean motion may be caused by the comet passing through a resisting medium in the vicinity of the *Sun*, and the variation of the acceleration may be due to the varying density of this medium. He also hinted at the possibility of the density of the medium varying with the sun-spot cycle, but admitted that there was little evidence upon which to base this assumption.

B. *Comet Pons-Winnecke*. TISSERAND mentions⁽⁴⁾ that two investigations of the *Pons-Winnecke Comet* similar to that of *Comet Encke* were made by OPPOLZER and BARON DE HAERDTL, in which the former found an acceleration of mean motion, but the other none.

C. *Comet Brooks*. Any case where a careful computation of perturbations has been made in order to represent three or more appearances of a comet might be called an investigation of this type. BAUSCHINGER⁽⁵⁾ in attempting to represent the observations of *Comet Brooks* for the appearances of 1889, 1896, and 1903, found that he could not represent all three by a single system of elements. However, by using the system of the last two appearances only, he predicted accurately the return of 1910 when the comet was seen but once. Later, after the comet was not seen in the return of 1918, DUBIAGO⁽⁶⁾ revised and extended the system to predict the return of 1925, but his T by this time was 5 days in error.

D. *Comet Kopff*. Recently KEPINSKI⁽⁷⁾ has succeeded in representing the observations of the last three appearances of *Comet Kopff* in 1919, 1926, and 1932-33 by a unique system of elements with a maximum normal residual of +6".82.*

IV. PREVIOUS WORK ON *Comet D'Arrest*.

It is not planned to make an exhaustive list of all the literature on *Comet d'Arrest*. Only the material immediately useful for this and future investigations will be mentioned.

Eight normal places for 1851 were derived by OUDEMANS⁽⁸⁾ and six normal places for 1857-58 by LIND⁽⁹⁾. Seven normal places in these two appearances are well represented by a system of elements determined by VILLARCEAU, who carried the calculation of perturba-

* Since this paper was written *Comet Kopff* has been found again (April 22, 1939) in exactly the place predicted by Kepinski.

tions past a very close approach of the comet to *Jupiter* in 1861, 0.36 A. U., to obtain a search ephemeris for 1863. Since 1863 the calculation of perturbations has been brought from each appearance to the next by LEVEAU until 1910. A solution combining the normal places for the first three appearances and the finding ephemeris for 1877 were published by LEVEAU in 1877.⁽¹⁰⁾ He also gave in the *Bulletin Astronomique*⁽¹¹⁾ a system of elements closely fitting normal places in 1890 and 1897 and predicting the return of 1910. Computations for the period 1910-17 were made by BRAAE⁽¹²⁾ and the return of 1923 was predicted by CRIPPS⁽¹³⁾ and by DUBIAGO and LEXIN⁽¹⁴⁾. The ephemeris for the return of 1930 was supplied by the Computing Section of the British Astronomical Association.⁽¹⁵⁾ The prediction for 1936-37 was made by FOXELL and LEVIN⁽¹⁶⁾. Table 1

TABLE 1.
OBSERVATION PERIODS OF *Comet d'Arrest*.

Appearance	T	First Obs.	Last Obs.	Total Days
1851	July 8	June 27	Sept. 30	95
1857-58	Nov. 28	Dec. 5	Jan. 18	44
1870	Sept. 22	Aug. 31	Dec. 20	112
1877	May 10	July 9	Sept. 10	62
1890	Sept. 18	Oct. 6	Dec. 13	68
1897	May 23	June 28	Oct. 3	97
1910-11	Sept. 16	Aug. 26	Jan. 27	154
1923-24	Sept. 15	Sept. 5	Jan. 31	148

$$\left\{ \begin{array}{l} -14^{\circ}00A - 14^{\circ}32B + 3^{\circ}34C + 0^{\circ}00063D - 8^{\circ}74E - 2^{\circ}52F = +170^{\circ}3 \\ +22^{\circ}00A + 22^{\circ}50B - 5^{\circ}25C - 0^{\circ}00100D + 13^{\circ}73E + 3^{\circ}97F = -214^{\circ}5 \\ -45^{\circ}60A - 47^{\circ}87B - 12^{\circ}29C + 0^{\circ}00107D - 46^{\circ}21E - 10^{\circ}43F = +577^{\circ}5 \\ \hspace{15em} + 20^{\circ}00F = -37^{\circ}7 \\ \hspace{10em} -20^{\circ}00C = -202^{\circ}2 \\ -20^{\circ}00A = -7^{\circ}7 \end{array} \right.$$

Here A, B, etc. are the desired coefficients for the equation of condition for the right ascension of this date. The numerical coefficients on the left are the changes made in the elements in the six arbitrary systems, and the constant terms on the right are the changes produced in the right ascension for this date by these changes in elements.

The terms containing A, C, and F are eliminated from the first three equations by substituting the values obtained from the last three equations, which leaves a system of three equations in three unknowns, the determinant of whose coefficients is

$$\begin{vmatrix} -14.32, +.00063, -8.74 \\ +22.50, -.00100, +13.73 \\ -47.87, +.00107, -46.21 \end{vmatrix} = +1.2838 - 1.2862 = -0.0024$$

lists the periods during which observations of *Comet d'Arrest* were made.

There have been a number of opportunities to make comparisons between former computations and the present one. Only two outstanding discrepancies have been discovered. A system beginning with VILLARCEAU's osculating elements of 1857 Dec. 25, when carried back to 1851, gave residuals for normal places there differing from those of VILLARCEAU by amounts up to 550". It is assumed that this difference is produced by VILLARCEAU's probable use of BOUVARD's value of 1/1070 for the mass of *Jupiter*, while this investigation employs the present accepted value of 1/1047.35.

Also it was found that there was a basic fallacy in the work of LEVEAU in the *Paris Memoires*⁽¹⁰⁾, in which he had apparently derived a system fitting normal places of the 1851, 1857-58, and 1870 apparitions with a maximum residual of 15", and predicted the return of 1877 within 160". LEVEAU determined the coefficients for his equations of condition of 1870 by applying arbitrary changes to VILLARCEAU's osculating elements of 1851, and determining the effects produced on the right ascensions and declinations for the normal dates in 1870. The changes in elements were so small that they would produce no appreciable changes in the perturbations except for the period 1859-63 when the comet came quite close to *Jupiter*. For this period LEVEAU re-computed the perturbations for his six arbitrary systems. For example, the effects on the right ascension of the normal date of 1870 Sept. 24 were as follows:

It is evident that the products of the coefficients have no accuracy beyond the third significant figure; therefore the value of this determinant is zero for all practical purposes, and the solution for B, D, and E becomes indeterminate. Since the solution must be made with the same determinant for the right ascensions and declinations of all the normal dates of 1870, all the values of B, D, and E will be indeterminate. It was found that the coefficients D, of the mean motion, as given by LEVEAU were about 300 times too large and of the wrong sign.

LEVEAU could have discovered this fallacy by actually recomputing the perturbations with his new system for the period 1859-63 but he depended upon the solution of his indeterminate equations of condition. His pre-

diction for 1877 would have been almost as close if he had left VILLARCEAU'S system of 1851 unchanged.

V. METHODS OF COMPUTATION.

At the outset many problems regarding methods of computation had to be decided. The original decisions were sometimes modified as experience was gained. Only those methods that were finally adopted will be described here.

A. Planetary Coordinates. Since the first integration scheme was begun for the year 1894 there were no planetary coordinates in x, y, z listed in *Planetary Coordinates 1800-1940*. For *Jupiter* and *Saturn* the longitude, latitude, and radius vector only are given from 1800 to 1900 for 100-day intervals referred to the equinox of 1950.0. The x, y, z and X, Y, Z values were obtained to 20-day and 10-day intervals as required. It was found, after actually working out tables for a year's period for *Venus*, *Earth*, and *Mars* by using ephemeris positions and bringing them up to 1950.0 that sufficient accuracy could be obtained by using the tables given for these planets for the period 1920-40. It was merely necessary to compute the 1950 longitude for a given date, to find some place in the tables which had a similar value, and to interpolate for the small difference, which remained nearly constant for several years. It was found later that even if the difference in date were 70 years the coordinates of these planets could be obtained by this method within one unit in the third decimal place, and the sun-planet attractions within a unit in the ninth decimal place. The coordinates of *Uranus* and *Neptune* for 1950 were calculated for 400-day intervals and interpolated for 80-day intervals. Most of the influence of *Mercury* was provided for by adding its mass to the *Sun*; while this is not rigorous it seemed accurate enough for this investigation, because the closest approach of *Comet d'Arrest* to the *Sun* is 1.2 A. U. *Pluto* was not considered.

B. Application of Cowell's Method. COWELL'S method with a ten-day interval has been used exclusively. Attractions are computed to the ninth decimal place and the sums rounded off to eight in the integration scheme. While a 20-day interval is adequate except near perihelion, twenty days seems too large a fraction of the period of *Venus* to give a good representation of the *Sun-Venus* attractions. Using the ten-day interval entirely makes unnecessary the change of interval on both sides of perihelion, and avoids chances of error. Of course, using a shorter interval makes it necessary to keep the ninth decimal place in the attractions accurate. For this reason, the *Uranus* and *Neptune* attractions—never more than a few units in the ninth place—are kept to the tenth place and accumulated in

each coordinate before adding to the other attractions, in order to avoid systematic trends. While it is rather difficult to use COWELL'S method near perihelion, even with a ten-day interval, the labor here can be reduced materially by calculating the x, y, z of the comet for ten or more intervals in advance by determining the A_x, B_x , etc., coefficients at a given point. After the integration has been carried through perihelion once, the differences can be used in approximating the attractions for another time.

The general procedure of applying perturbations by COWELL'S method is first to determine by interpolation the sun-planet attractions for the ten-day intervals. This is generally done for many dates at one time. (The Nautical Almanac Office could greatly facilitate this part of the work by publishing the barycentric coordinates of the *Sun* for the ten-day intervals.) Next the comet-planet attractions are calculated, for most of the planets at 20-day intervals except at close approaches, and the values are interpolated for the intermediate dates as the integration scheme proceeds. For *Mars* the intervals of exact calculation are 40 days, and for *Uranus* and *Neptune*, 80 days. The sums of the sun-planet and comet-planet attractions are checked by differencing before adding to the sun-comet attractions for each ten-day interval. To maintain ninth-decimal accuracy for the sun-comet attractions Table X of *Planetary Coordinates* has not been used when $r^2 < 20$; instead the square root has been taken, and the additional time required to obtain the sun-comet attractions in this way is only about a minute for each step. The double summation in the integration scheme is done without the machine first and checked with the machine. A check computation is always made which repeats the addition of the planetary attractions and the computation of the sun-comet attractions, and keeps an independent integration scheme.

C. Accuracy of the Method. In order to retain all accuracy possible in the eight-place numerical integration, when it was necessary to go into and out of the integration schemes, the BAUSCHINGER-PETERS eight-place logarithmic tables were used to obtain the values of the sines and cosines of the geometric elements, and to make the final approximations for $\sin E$ in each solution of KEPLER'S equation.

After BROUWER published his paper⁽¹⁷⁾ indicating that the cumulative error of numerical integration varies as $n^{\frac{3}{2}}$, where n is the number of steps of integration, the 1851 July 6.5 integration scheme was carried forward 2400 days without applying perturbations, using eight decimals and ten-day intervals. By the end of this period the following changes had been produced in the

elements:

$$\begin{array}{ll} \Delta\Omega' - 0''.9 & \Delta e + .0000002 \\ \Delta i' + 0''.2 & \Delta a + .000012 \\ \Delta\omega' + 0''.7 & \Delta T + {}^d.00308 \end{array}$$

An approximate comparison of the method of numerical integration in computing perturbations and the older method of variation of elements is afforded by the re-computation in this investigation of the period 1890-1910, using LEVEAU's original elements. After twenty years the integration gives very small differences in all the elements except T , which is earlier than LEVEAU's

T by .009 days. Applying BROUWER's $n^{\frac{1}{2}}$ rule the accumulated error for twenty years in this case should be about +.015 days. The difference of .024 days might well be explained by the fact that LEVEAU does not compute the influence of the inner planets except when the comet is in the vicinity of perihelion.

In any case the error introduced by the method of numerical integration can be only a small fraction of the large systematic changes in elements which are discussed in later sections.

D. *Samples of Computation.*

1. PORTION OF INTEGRATION SCHEME IN X

Date, 1891	$''f(x)$	$'f(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f''''(x)$
Jan. 17.5	+1.46303660		-636017				
		-2996796		+79280			
Jan. 27.5	+1.43306864		-556757		-9224		
		-3553553		+70056		+802	
Feb. 6.5	+1.39753311		-486689		-8422		+63
		-4040242		+61634		+865	
Feb. 16.5	+1.35713069		-425047		-7557		-9
		-4465289		+54077		+856	
Feb. 26.5	+1.31247780		-370970		-6701		-39
		-4836259		+47376		+817	
Mar. 8.5	+1.26411521		-323594		-5884		

2. PLANETARY ATTRACTIONS IN X (Units of 7th decimal place.).

Date 1891	Feb. 16.5	Feb. 26.5	Mar. 8.5	Mar. 18.5
Differences	-.12	-.06	-.13	
	-.21	-.33	-.39	-.52
Sum of Planetary Attractions	-1.58	-1.91	-2.30	-2.82
<i>Sun—Jupiter</i>	-9.25	-9.36	-9.46	-9.57
<i>Sun—Saturn</i>	+ .93	+ .93	+ .93	+ .93
<i>Sun—Mars</i>	- .03	- .03	- .02	- .02
<i>Sun—Earth</i>	+ .78	+ .85	+ .89	+ .90
<i>Sun—Venus</i>	+1.36	+1.20	+ .96	+ .64
<i>Sun—Uranus, Neptune</i>	+ .01	+ .01	+ .01	+ .01
<i>Comet—Uranus, Neptune</i>				
<i>Comet—Jupiter</i>	+5.66	+5.50	+5.37	+5.25
<i>Comet—Saturn</i>	- .77	- .78	- .78	- .79
<i>Comet—Mars</i>	- .07	- .06	- .06	- .05
<i>Comet—Earth</i>	- .13	- .11	- .09	- .08
<i>Comet—Venus</i>	- .07	- .06	- .05	- .04

3. COMPUTATION OF 80-DAY INTERVAL. 1891 Mar. 8.5

"f(x,y,z) - 1/240 f''(x,y,z) 1/12 f(x,y,z)	+1.26411546 -26966	+1.86965470 -39882	+ .14350006 -3060	5.11200724	r ²
x ₀ , y ₀ , z ₀	+1.26384580	+1.86925588	+ .14346946		
Sun—Comet Attractions	-32357.12	-47856.89	-3673.12		
Planetary Attractions	-2.30	-2.00	+ .50		
f(x,y,z)	-32359.42	-47858.89	-3672.62		
Jupiter (x,y,z)	+4.2359	-2.4354	-1.1486	29.033	ρ ² Table X
Comet (-x ₀ , -y ₀ , -z ₀)	-1.2638	-1.8693	- .1435	.20214	
(x-x ₀), (y-y ₀), (z-z ₀)	+2.9721	-4.3047	-1.2921		
Saturn	- 9.024 - 1.264	+ 2.128 - 1.869	+ 1.269 - .143	107.18 .02850	
K 2.674	-10.288	+0.259	+ 1.126		
Uranus	-16.1 - 1.3	- 8.5 - 1.9	- 3.5 - .1	423.9 .1146	
.0129	-17.4	-10.4	- 3.6		
Neptune	+11.7 - 1.3	+25.5 - 1.9	+10.1 - .1	765.1 .0472	
.0153	+10.4	+23.6	+10.0		
Mars	+ .794 -1.264	+1.157 -1.869	+ .510 - .143	.862 .0395	
3.03	- .470	- .712	+ .367		
Earth	- .975 -1.264	+ .174 -1.869	+ .076 - .143	7.89 .0451	
.897	-2.239	-1.695	- .067		
Venus	- .501 -1.264	- .488 -1.869	- .188 - .143	8.78 .0384	
.728	-1.765	-2.357	- .331		

E. *Combination of Appearances.* Whenever a solution involving one or more appearances was made to improve the representation of normal places the coefficients of the equations of condition were computed in the form given by STRACKE'S *Bahnbestimmung* (page 327) for elliptic orbits, unless there had been a close approach to *Jupiter* in the period involved. In the latter case

coefficients were derived by making arbitrary changes in the elements, recomputing the perturbations for the period of close approach, and determining directly the effects of these changes on the right ascensions and declinations for the normal dates.

The solutions of these equations of condition were made by the method of least squares, the equations

each being given unit weight. It was intended to make final solutions with the weighted equations in order to obtain the best fit possible for a single system of osculating elements which would represent closely the observations of all eight appearances of *Comet d'Arrest*. Since such a single system could not be found weighted solutions were not made in the present investigation.

VI. DERIVATION OF NORMAL PLACES.

The normal places of VILLARCEAU were retained for the appearances of 1851 and 1857-58, and those of LEVEAU for 1870. For four of the other five appearances normal places had been used by investigators, but their values were not published, and normal places had to be derived again for all five appearances. This was done by reducing each observation anew, accepting only the observer's time and his measures of differences in right ascension and declination from the comparison stars. The best catalogue positions were used for the comparison stars, and in case a faint star was used for the direct comparison and afterward connected with some brighter star, the position of the fainter star was obtained from the *Carte du Ciel*. Whenever possible, the proper motion of the comparison star was taken into account. The *Geschichte des Fixstern-Himmels* could be used for the observations of 1877 and 1897 only, where the declinations were positive.

The observations of BARNARD and STONE in 1890, of PERRINE with the Lick 36-inch refractor in 1897, and of VAN BIESBROECK—two with the Yerkes 40-inch and nine photographic—and of the Franklin-Adams Star Camera at Johannesburg—except the two pre-discovery positions—in 1923, were given double weight. Except in 1877, when observations were few, observations were rejected if they differed by more than 10" from the normal. The normal places of 1890, 1897 and 1910-11 were derived by the use of ephemerides that already closely approximated the observations. All the normal places used in this investigation are listed in Table 2. The sources of observations used in deriving normal places for the last five appearances are given in Table 3.

VII. GENERAL PROCEDURE OF THE INVESTIGATION.

The first integration scheme was started by using LEVEAU's osculating elements of 1894 Dec. 31.0 (Berlin), and the calculation of perturbations was carried back to 1890 and ahead to 1897, the representation of observations agreeing within 15" of that of LEVEAU. By means of this system normal places for the observations of 1890 and 1897 were derived.

A new integration scheme was started at 1910 September 20.5 G. M. T., the basis for which was a solution of a system of equations of condition involving normal

TABLE 2.
NORMAL PLACES AND WEIGHTS.

Year	Date	G.M.T.	α 1950.0	Wt.	δ 1950.0	Wt.
1851	July	2.96279	15°19459	16	+11°30339	15
	July	30.96279	42.38490	23	+ 9.54223	23
	Aug.	28.96279	60.53613	28	+ 4.20971	28
	Sept.	28.96279	67.19951	26	- 2.94617	26
1857	Dec.	11.96279	306.54716	12	-20.63985	12
	Dec.	28.96279	324.81452	12	-18.23783	7
1858	Jan.	13.96279	340.80793	13	-14.56108	12
1870	Sept.	23.96279	269.62425	15	-20.74664	16
	Oct.	17.96279	294.44421	14	-26.82883	14
	Nov.	18.96279	327.71567	9	-25.52851	9
	Dec.	12.96279	347.96667	6	-19.96981	8
1877	July	9.5	43.99183	7	+ 7.22061	7
	Aug.	8.5	60.54850	4	+ 6.40872	2
	Sept.	7.5	70.90296	1	+ 3.28581	1
1890	Oct.	9.5	292.10901	37	-26.53978	41
	Nov.	8.5	322.12911	39	-26.96853	44
	Dec.	8.5	346.95101	12	-20.80875	12
1897	July	4.5	35.00289	19	+ 6.68790	28
	Aug.	3.5	53.39389	29	+ 6.03119	27
	Sept.	2.5	65.22061	16	+ 2.67511	17
	Oct.	2.5	68.76830	4	- 2.28742	4
1910	Aug.	26.5	252.72111	8	- 9.83701	7
	Sept.	25.5	277.69379	16	-24.13404	15
	Oct.	30.5	316.02862	25	-29.38601	25
	Nov.	24.5	333.73254	26	-25.03923	26
	Dec.	24.5	1.34052	10	-16.60712	12
1911	Jan.	18.5	15.74934	7	- 9.49464	7
1923	Sept.	7.5	265.5511	2	-16.6598	2
	Nov.	26.5	336.54146	4	-26.20192	4
	Dec.	11.5	347.53681	5	-22.71711	5
1924	Jan.	5.5	3.33162	14	-16.10614	16
	Jan.	30.5	16.97771	8	- 9.56619	8

places in 1897 and three approximate normal places in 1910, accepting LEVEAU's values for the perturbations for the period 1897-1910. This new system represented quite closely the observations of 1910-11, and was used to derive six normal places. The calculation of perturbations by this integration scheme was carried forward to 1924, where it was used to establish normal places for the observations of 1923-24, and back to 1890. When

TABLE 3.
OBSERVATIONS USED IN DERIVING NORMAL PLACES

Year	Place	Observer	No. Obs.	References
1877	Athens	Schmidt	5	<i>A. N.</i> 90; 165, 173, 191, 367.
	Leipzig	Peters	2	<i>A. N.</i> 91; 300.
	Marseilles	Coggia	1	<i>A. N.</i> 90; 87.
	Milan	Schiaparelli	4	<i>A. N.</i> 90; 223.
1890	Charlottesville	Stone	9	<i>Astr. Jour.</i> 10; 126, 135, 167.
		Parrish	9	
		De Mott	3	
	Mt. Hamilton	Barnard	26	<i>Astr. Jour.</i> 11; 6.
	Munich	Bauschinger	2	<i>A. N.</i> 125; 383, <i>A. N.</i> 126; 31.
	Padua	Abbeti	9	<i>A. N.</i> 126; 79.
	Paris	Bigourdan	1	<i>C. R.</i> 111; 521.
	Rome	Millosevich	3	<i>A. N.</i> 125; 381, <i>A. N.</i> 126; 31.
	Strassburg	Kobold	3	<i>A. N.</i> 125; 381, <i>A. N.</i> 126; 230.
	Vienna	Palisa	1	<i>A. N.</i> 125; 383.
	Washington	Frisby	6	<i>Astr. Jour.</i> 10; 99.
	Windsor, N. S. W.	Tebbutt	11	<i>A. N.</i> 126; 357.
1897	Algiers	Rambaud	2	<i>C. R.</i> 125; 83.
		Sy	2	
	Besançon	Chofardet	5	<i>A. N.</i> 146; 299.
		Petit	5	
	Liverpool	Plummer	1	<i>M. N.</i> 59; 101.
	Mt. Hamilton	Perrine	30	<i>Astr. Jour.</i> 17; 192.
		Aitken	2	
	Rio de Janiero	Cruls	3	<i>A. N.</i> 146; 91.
	Teramo	Cerulli	2	<i>A. N.</i> 145; 330.
	Toulouse	Rossard	5	<i>C. R.</i> 125; 82, 372.
	Washington	Brown	8	<i>Astr. Jour.</i> 18; 79.
1910-11	Algiers	Gonnessiat	11	<i>Bull. Astron.</i> 28; 273.
		Rambaud	22	
	Nice	Javelle	34	<i>Bull. Astron.</i> 29; 33.
	Santiago	Castro	20	<i>A. N.</i> 194; 33.
		Devoto	2	
	Washington	Eppes	1	<i>U. S. Naval Obs.</i> , 2nd Series XII; 188.
	Williams Bay	Barnard	11	<i>Astr. Jour.</i> 41; 153.
1923-24	Bergedorf	Baade and Graff	2	<i>B. Z.</i> 1924 Nr. 2, 4.
	Besançon	Chofardet	1	<i>Astr. Jour.</i> 35; 188.
	Capetown	Reid	1	<i>A. N.</i> 220; 79.
	Johannesburg	Franklin-Adams Star Camera	9	<i>Union Obs. Circular</i> 60.
		Wood, Worsell, Innes	29	
	Williams Bay	Van Biesbroeck	11	<i>Astr. Jour.</i> 35; 154.

it was found that this system could not be modified so as to give a good representation of the normal places of the whole period 1890-1924 in time to furnish a prediction

for the 1936 return, attention was shifted to the first four appearances.

Had this integration scheme been continued backward

the residuals, already large for 1890, would have become excessively large for the earlier apparitions. Consequently a new integration scheme was started with VILLARCEAU's osculating elements of 1857 Dec. 25.0 (Berlin) and this was used in the calculation of perturbations back to 1851 and ahead to 1890.

To keep chronological order the system for the period 1851-1890 has been called System I, and that for 1890-1924, System II. The osculating elements for each apparition and the residuals of the normal places for these two main systems are given in Table 4.

After these two main systems had been established efforts were made to improve the representation of three and finally two appearances at a time by least-squares solutions of the equations of condition, in the hope that a single system could be found to represent all eight appearances acceptably. The results of these efforts are described in the next two sections.

VIII. COMBINATIONS OF THREE APPEARANCES.

A. 1890, 1897, 1910-11. Early attempts were made to find a single system involving these well-observed appearances in order to predict more accurately the

return of 1937. Since there was a relatively close approach of the comet to *Jupiter* in May, 1908, (1.3 A. U.), it was necessary to use the method of making arbitrary changes in each element separately to obtain differential coefficients for the equations of condition. A least-squares solution of the twenty-six equations of condition reduced residuals from 1900" to 125", but only by making very large changes in the geometric elements—up to 300" in the case of ω' . The impossibility of representing the normal places with any degree of accuracy indicated that some unaccountable change in the orbital elements had taken place during this period.

B. 1877, 1890, 1897. This is the only series of three successive appearances of *Comet d'Arrest* that does not involve a close approach of the comet to *Jupiter*. For this reason there is less chance of error here in evaluating the differential coefficients of the equations of condition, and even if the changes of elements required by the solution are large, the resulting changes in perturbations would be minor.

It was found that to obtain even an approximate fit of normal places large changes were required in Ω' , ω' ,

TABLE 4.
OSCULATING ELEMENTS AND RESIDUALS, SYSTEMS I AND II.

System I (1851-1890)						
G.M.T. Date of Osculation		1851 July 6.5	1857 Dec. 21.5	1871 Jan. 2.5	1877 Aug. 8.5	1890 Nov. 8.5
1950.0	Ω'	31°73297	31°81752	39°71315	40°14016	40°17954
	i'	13°30221	13°30595	13°10308	13°18366	13°19200
	ω'	294°00558	293°95313	281°79622	281°73282	281°69241
	e	.6593197	.6598733	.6348308	.6276635	.6273206
	a	3.444192	3.439865	3.506495	3.540596	3.545802
	μ°	°1541959	°1544870	°1501047	°1479413	°1476156
	M	359°68837	3°60150	15°30119	13°36652	7°82265
	T	July 8.52103	Nov. 28.18735	Sep. 22.56322	May 10.14983	Sep. 17.50660

System II (1890-1924)					
G. M. T. Date of Osculation		1890 Nov. 8.5	1897 Sept. 2.5	1910 Sept. 20.5	1923 Nov. 26.5
1950.0	Ω'	40°19522	40°10695	40°52095	50°88913
	i'	13°19026	13°19018	13°21214	13°65732
	ω'	281°65610	281°74410	281°94910	269°17393
	e	.6274793	.6266239	.6369548	.6156434
	a	3.546655	3.551417	3.497887	3.532082
	μ°	°1475624	°1472657	°1506591	°1484766
	M	7°74140	15°08131	0°61916	10°81519
	T	Sep. 17.03815	May 23.09116	Sep. 16.39032	Sep. 14.65895

Residuals of System I					Residuals of System II				
Normal Places		O-C		$\Delta\delta$	Normal Places		O-C		$\Delta\delta$
		$\Delta\alpha \cos \delta$					$\Delta\alpha \cos \delta$		
1851	July 3	- 554".4		+ 66".9	1890	Oct. 9.5	-1891".2		+511".2
	July 31	- 386.9		+ 9.8		Nov. 8.5	-1695.4		+106.5
	Aug. 29	- 300.6		- 92.3		Dec. 8.5	-1285.3		-126.5
	Sep. 29	- 338.8		- 188.0					
1857	Dec. 12	- 0.6		+ 1.8	1897	July 4.5	- 355.5		- 10.6
	Dec. 29	+ 0.9		+ 2.8		Aug. 3.5	- 279.0		- 36.3
1858	Jan. 14	+ 0.9		+ 0.0		Sep. 2.5	- 247.8		- 79.1
						Oct. 2.5	- 270.3		-125.1
1870	Sep. 24	- 410.1		+ 134.3	1910	Aug. 26.5	+ 2.9		+ 2.3
	Oct. 18	- 428.6		+ 102.0		Sep. 25.5	+ 3.9		+ 3.6
	Nov. 19	- 370.1		- 5.8		Oct. 30.5	+ 0.1		+ 1.7
	Dec. 13	- 283.3		- 47.2		Nov. 24.5	+ 3.0		+ 0.2
1877	July 9.5	- 381.9		- 26.8		Dec. 24.5	+ 1.4		- 0.8
	Aug. 8.5	- 320.0		- 47.6	1911	Jan. 18.5	+ 3.4		+ 3.3
	Sep. 7.5	- 300.4		- 60.0					
1890	Oct. 9.5	-3837		+1021	1923	Sep. 7.5	-1388		+576
	Nov. 8.5	-3400		+ 200		Nov. 26.5	- 961.6		+ 5.8
	Dec. 8.5	-2564		- 265		Dec. 11.5	- 817.7		- 64.3
					1924	Jan. 5.5	- 619.2		- 94.7
						Jan. 30.5	- 480.2		- 86.9

and a . The best representation still left residuals up to 100", and the results of the solution were checked by actually recomputing the perturbations for this period with the changed elements. Consequently, for this period, too, there seems indication of an unaccountable change of elements.

C. 1890, 1897, 1910-11. Computations of the perturbations for the period 1851-58 with the modern accepted value of the mass of *Jupiter* instead of the value used by VILLARCEAU indicated how great a change in the representation could be produced by a change in the mass of *Jupiter*, even in a period throughout which *Jupiter* was a long distance from the comet.

I thought it would be of interest to find out if a small change in the mass of *Jupiter*, which is predominately responsible for the perturbations of *Comet d'Arrest*, might make it possible still to represent the normal places of 1890-97 and 1910-11 with one system of osculating elements. Although the probable error of the various determinations of the mass of *Jupiter* ⁽¹⁸⁾ indicates that it has been accurately determined, it was felt that this investigation could not be complete without testing the effect of a change in the mass of *Jupiter*.

Accordingly additional differential coefficients for the

equations of condition were obtained by making a change of 0.16% in the mass of *Jupiter* and recalculating the perturbations for the 1890-1910 period. It was found that this change in *Jupiter's* mass changed the representation by amounts up to 137". However, a new solution of the equations of condition required a preposterous change of 0.65% in the mass of *Jupiter*, and still left residuals up to 60". The representation could have been similarly improved by introducing almost any seventh variable. Consequently, the changes in the comet's orbital elements for this period remain still unaccounted for.

IX. COMBINATIONS OF TWO APPEARANCES.

In order to determine which elements were apparently undergoing change the successive appearances were considered two at a time. Thus were formed seven systems of osculating elements which gave the best representation possible of the normal places for the pairs of successive appearances. All but the 1851 and 1923-24 appearances are represented in two consecutive systems; and the osculating elements for the six remaining appearances can be compared in successive systems to get some idea of what elements appear to be changing. Table 5 shows

TABLE 5.
SYSTEMS OF APPEARANCES IN PAIRS

Osc. Date G.M.T.	Elements, Equinox 1950.0		Normal Dates	Representation	Changes of Elements Successive Systems
1851 July 6.5	System 1851-57			System 1851-57	
	Ω' i' ω' e T^a	31°73669 13°30169 294°00230 .6593004 3.444039 July 8.67738		O-C $\Delta\alpha \cos \delta$ $\Delta\delta$ 1851 July 3 July 31 Aug. 29 Sep. 29 -0°6 -5°1 -0.4 +2.5 +5.8 +3.9 -3.2 -5.1	
1857 Dec. 21.5	System 1857-70			System 1857-70	
		31°82127 13°30536 293°94985 .6598540 3.439712 Nov. 28.18722			
		31°82429 13°30578 293°93982 .6598627 3.440020 Nov. 28.17961			
		1857 Dec. 12 Dec. 29 1858 Jan. 14		-1°9 +1°7 +0.7 +3.5 -5°8 +2°2 +0.5 +4.2 -5.1 +1.4 +5.2 +2.9	$\Delta\Omega' + 10''9$ $\Delta i' + 1''5$ $\Delta\omega' - 36''1$ $\Delta e + .0000087$ $\Delta a + .000308$ $\Delta T - d.00761$
1871 Jan. 2.5	System 1870-77			System 1870-77	
		39°72294 13°10325 281°78724 .6348652 3.506446 Sep. 22.69491			
		39°71772 13°10448 281°78478 .6348923 3.506698 Sep. 22.68662			
		1870 Sep. 24 Oct. 18 Nov. 19 Dec. 13		-2°5 -7°7 +3.7 +5.1 -3.5 -0.2 +1.0 -5.4 -0°4 -3°2 +1.8 +5.8 -3.5 -4.2 +3.6 -10.0	$\Delta\Omega' - 18''8$ $\Delta i' + 4''4$ $\Delta\omega' - 8''9$ $\Delta e + .0000271$ $\Delta a + .000252$ $\Delta T - d.00829$
1877 Aug. 8.5	System 1877-90			System 1877-90	
		40°14342 13°18496 281°72210 .6277336 3.540764 May 10.44277			
		40°14226 13°18456 281°71563 .6277734 3.540973 May 10.44497			
		1877 July 9.5 Aug. 8.5 Sep. 7.5		+4°6 +4°0 -7.2 +7.5 -15.0 +33.2 +2°8 +3°0 -4.9 +6.7 +2.8 +36.3	$\Delta\Omega' - 4''2$ $\Delta i' - 1''4$ $\Delta\omega' - 23''3$ $\Delta e + .0000398$ $\Delta a + .000209$ $\Delta T + d.00220$
1890 Nov. 8.5	System 1890-97			System 1890-97	
		40°18199 13°19294 281°67525 .6274332 3.546157 Sep. 17.59417			
		40°19517 13°19026 281°66459 .6274510 3.546345 Sep. 17.59665			
		1890 Oct. 9.5 Nov. 8.5 Dec. 8.5		+1°1 +2°1 -0.2 -4.7 -0.4 -6.3 +1°0 -2°6 -0.3 -1.2 -2.4 +1.7	$\Delta\Omega' + 47''4$ $\Delta i' - 9''6$ $\Delta\omega' - 38''4$ $\Delta e + .0000178$ $\Delta a + .000188$ $\Delta T + d.00248$
1897 Sep. 2.5	System 1897-1910			System 1897-1910	
		40°10690 13°19017 281°75259 .6265956 3.551106 May 23.33091			
		40°10695 13°18949 281°75102 .6266163 3.551306 May 23.33716			
		1897 July 4.5 Aug. 3.5 Sep. 2.5 Oct. 2.5		-0°9 +4°3 +1.5 +1.4 +0.8 -0.8 -0.7 -1.4 -4°2 +2°5 -0.8 0.0 +2.5 -1.9 +3.1 -2.4	$\Delta\Omega' + 0''2$ $\Delta i' - 2''4$ $\Delta\omega' - 5''7$ $\Delta e + .0000207$ $\Delta a + .000200$ $\Delta T + d.00625$
1910 Sep. 20.5	System 1910-23			System 1910-23	
		40°52046 13°21145 281°95565 .6369441 3.497795 Sep. 16.39704			
		40°51959 13°20826 281°95846 .6369906 3.498069 Sep. 16.39909			
		1910 Aug. 26.5 Sep. 25.5 Oct. 30.5 Nov. 24.5 Dec. 24.5 1911 Jan. 18.5		-3°7 -1°1 +4.2 -0.4 +1.4 +1.3 +1.3 +1.0 -3.8 -0.8 -4.0 +2.2 -17.6 -0.1 +9°4 -26°0 +12.2 -17.2 -2.5 -4.8 -9.0 -3.1 -17.1 -4.0	$\Delta\Omega' - 3''1$ $\Delta i' - 11''5$ $\Delta\omega' + 10''1$ $\Delta e + .0000465$ $\Delta a + .000274$ $\Delta T + d.00205$
1923 Nov. 26.5	System 1923-24			System 1923-24	
		50°88581 13°65316 269°18971 .6157098 3.532191 Sep. 15.07823			
		1923 Sep. 7.5 Nov. 26.5 Dec. 11.5 1924 Jan. 5.5 Jan. 30.5		-22°3 +39°9 +28.1 +0.7 +25.1 -6.8 +16.3 +1.1 -0.5 +7.2	

these systems, their representation of the normal places, and the apparent changes in the elements. It is plainly evident here why it was impossible to represent the normal places of three consecutive appearances with a single system of osculating elements.

X. CONCLUSIONS.

In the Introduction were listed six possible causes for the differences between the observed and computed positions of a periodic comet. The intention here is to consider these separately and to determine which of them are revealed by the present investigation as operative in the case of *Comet d'Arrest*.

(a) *Comet d'Arrest*, like most other comets of *Jupiter's* family, is very diffuse. While accidental errors in micrometer settings should be eliminated for normal places derived from a large number of observations, the diffuseness of the comet must add uncertainty to normal places derived from only a few observations.

(b) Although early investigators complain about systematic errors in observations, these seem to be limited to observatories rather than periods, and could hardly be used as evidence of oscillatory motion of the nucleus. The observations of *d'Arrest* are probably not exact enough nor numerous enough to determine such cyclic motion, if it exists. If the center of light of *Comet d'Arrest* were not its center of gravity a systematic error of observations would be produced at each appear-

ance, but it is hard to understand how this could produce a cumulative and consistent change in the elements.

(c) Perhaps the greatest factor operating for indeterminateness of elements in the case of *Comet d'Arrest* is the short arc over which it is observed at each appearance. VAN BIESBROECK's computation of the definitive orbit of *Comet Geddes*⁽¹⁹⁾ shows that even with modern exact observations and an arc embracing more than two years of the comet's motion, the probable errors of the final elements are still rather large. In the case of *Comet d'Arrest* the longest continuous period of observation is five months during 1910-11—less than one-fifth the period for *Comet Geddes*. The indetermination of elements for *Comet d'Arrest* must, of course, be correspondingly higher. To test this a least-squares solution of the equations of condition of 1910-11 alone was made to improve the representation given by System II. The first solution gave reasonable changes for all the elements save a , for which the indicated change was so large that if it were carried forward to 1923 it would increase the residuals there by as much as 4000". A second solution was made without the coefficients for da and gave almost as good a representation, showing that the value of a cannot be determined accurately from a single apparition of *Comet d'Arrest*—even the best one. The results of these solutions are shown below:

Changes to Elements of System II		Normal Places	Residuals (O—C)					
First Solution (All elements)	Second Solution (Without a)		System II		First Solution		Second Solution	
		1910	$\Delta a \cos \delta$	$\Delta \delta$	$\Delta a \cos \delta$	$\Delta \delta$	$\Delta a \cos \delta$	$\Delta \delta$
$d\Omega' - 5''.9$	$-3''.1$	Aug. 26.5	+3''.0	+2''.3	-0''.7	-0''.6	-1''.0	-0''.4
$di' - 2''.2$	$-1''.9$	Sep. 25.5	+3.9	+3.6	+1.4	+1.1	+1.9	+0.9
$d\omega' + 11''.3$	$+9''.7$	Oct. 30.5	+0.1	+1.7	-1.2	+0.1	-1.3	+0.1
$de - .0000553$	$- .0000002$	Nov. 24.5	+3.0	+0.2	+1.5	-0.8	+1.1	-1.0
$da - .000559$		Dec. 24.5	+1.4	-0.8	-0.9	-1.8	-1.1	-1.8
$dT - .000050$	$+ .000158$	1911						
		Jan. 18.5	+3.4	+3.3	+0.1	+2.0	+0.5	+2.3

(d) Since accurate special perturbations have been computed throughout this investigation, any uncertainty in the elements cannot be due to approximate computation of perturbations.

(e) While the masses of the planets used in computing perturbations may be in error, these errors would have to improbably large for any planet in order to account for the apparent changes in the elements. An attempt to adjust the mass of *Jupiter* so as to represent the appearances of 1890, 1897, and 1910-11 (Section VIII) gave unsatisfactory results. Wherefore it seems likely

that inaccuracy in the masses of the planets could at most account for only a small portion of the apparent discrepancies in the motion of this comet.

(f) It seems clearly evident from Table 5 that some unknown force is operating to produce a consistent change in the dynamic elements of *Comet d'Arrest*. There are also apparent changes in the geometric elements but these, except ω' , show little consistency. The large changes in Ω' and ω' may be explained by the fact that the differential coefficients for these two elements in α are nearly alike and differ only in δ , and

also by the fact that large changes in these elements are required in the solutions to make up for discrepancies in a . The changes in all elements, except a , are smallest for 1870, 1897, and 1910-11 during which there were more accurate observations; having four or six normal places there must also heighten the accuracy. Inaccuracy on both sides of 1877 may be due to poor observations in 1877, for which only two normal places were used. The third date, Sept. 7.5, is the result of a single observation by SCHMIDT of ATHENS, and was not used in the least-squares solutions because the observer was not sure of his declination.

The most remarkable thing shown in this compilation is the large systematic change required in a in successive systems. It should be noted here that the O-C residuals in right ascension from finding ephemerides for *Comet d'Arrest* have always been large and negative, showing that the comet has always moved more slowly than predicted. This tendency is verified by the consistent magnification of a that is required to fit the normal places of successive apparitions. Further evidence that there are changes in elements beyond the requirements of the law of gravitation is that in nearly every solution involving equations of condition made in previous investigations or the present one, the final residuals show systematic runs instead of random distribution. As long as such systematic trends exist there is evidence of a forced fit and an indication that the motion of the

comet cannot be adequately represented by permitted changes alone of the six elements, and that the observed places are more accurate than they can be represented by any single set of osculating elements. Besides the change in a there appears also to be a consistent change required in e , and perhaps in T , although this is hard to separate from the change in T resulting from the change in a . To show these changes in a and e more clearly they are repeated in Table 6. Column 4 of this table requires explanation. If the change in elements be considered uniform, the elements given for the system 1851-57 will be the mean value for this period, or their value for 1854. Similarly those for the system 1857-70 will be approximately their value for 1863. Therefore the apparent changes in a and e between these systems are for the period 1854 to 1863, that is, $1\frac{1}{2}$ revolutions.

Apparently the elements which require empirical adjustment for *Comet d'Arrest* are the same that were changed in *Comet Encke*. However, in the case of *d'Arrest* the changes required are opposite in sign. If equations similar to those of BACKLUND are set up to represent the changes in mean motion and eccentric angle, taking the average apparent change over the 73-year period of observation of *Comet d'Arrest*, they are as follows:

$$\begin{aligned} n &= n_0 - ".035 \tau \\ \varphi &= \varphi_0 + 4".5 \tau \quad (\tau = t/2400) \end{aligned}$$

(2400 days is the approximate period of *Comet d'Arrest*)

TABLE 6.

CHANGES, a AND e ; SYSTEMS OF APPEARANCES IN PAIRS.

System	Δa	Δe	Interval in Revolutions
1851-57			
	+ .000308	+ .0000087	$1\frac{1}{2}$
1857-70			
	+ .000252	+ .0000271	$1\frac{1}{2}$
1870-77			
	+ .000209	+ .0000398	$1\frac{1}{2}$
1877-90			
	+ .000188	+ .0000178	$1\frac{1}{2}$
1890-97			
	+ .000200	+ .0000207	$1\frac{1}{2}$
1897-1910			
	+ .000274	+ .0000465	2
1910-23			
Totals	+ .001431	+ .0001606	$9\frac{1}{2}$
Ave. Change per Revolution	+ .000151	+ .0000169	1

In the present investigation no attempt has been made to discover whether the application of a uniform change in a and e would make it possible to represent the normal places within the accuracy of observation. This will have to be done, with the last four appearances of *Comet d'Arrest* at least, in order to approximate more closely the time and place of its return in 1943.

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